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(54) **Method and apparatus for interpolating between data samples**

Verfahren und Gerät, um zwischen Datenproben zu interpolieren

Procédé et appareil d'interpolation entre échantillons de données

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- **PROCEEDINGS OF THE IEEE vol. 61, no. 6, June 1973, pages 692 - 702; R.W SCHAFER et al.: "A digital signal processing approach to interpolation"**

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Description

Technical Field

The invention relates generally to preserving the frequency information of a sampled analog signal and, more particularly, to a method and apparatus for interpolating between data samples such that the interpolated values contain the same frequency domain information contained in the sampled analog signal.

Background of the Invention

Analog signals contain frequency domain information and time domain information that may be of interest to persons analyzing the signals. For various reasons, analog signals are often sampled by a time-based sampling process prior to analyzing the signal. Data samples produced by the sampling process contain both time domain information and frequency domain information about the analog signal. The time domain information, such as amplitude, for example, can be analyzed by measuring the amplitude of the data samples with an appropriate measuring device, such as a voltmeter, for example. Likewise, the frequency domain information can be analyzed by an appropriate apparatus, such as a digital spectrum analyzer, for example.

In order to find time and frequency domain information for points between data samples, an interpolation technique must be used. The prior art offers numerous time-based methods of interpolation that provide varying degrees of accuracy between the interpolated values and the analog signal. Like the original data samples, time domain information contained in an interpolated value, such as amplitude, can then be measured by an appropriate device, such as a voltmeter.

Likewise, if frequency domain information is sought for points between data samples, the data samples must be interpolated to find these values. Unfortunately, a time-based sampling process that produces data samples of an analog signal also produces spectral images of the analog signal's frequency spectrum. As a result, the data samples contain frequency domain information that includes the analog signal frequency spectrum information plus image information. The image information corrupts the frequency spectrum information so that the frequency domain information contained in the data samples does not accurately replicate the frequency spectrum of the analog signal. Prior art interpolation techniques further corrupt the frequency information contained in the data samples. As a result, the frequency information contained in the interpolated values does not accurately represent the frequency information contained in the analog signal.

In Schafer et al, "A Digital Signal Processing Approach to Interpolation," Proceedings of the IEEE, Vol. 61, No. 6, pages 692-702 (June 1973), the authors describe a prior art interpolation system for resampling an input signal at a uniformly spaced resample times. The process involves convolving the input samples with a filter impulse response at the resample times and summing. Since the input and output sample rates are related by a constant, the value of the filter impulse response need only be known at a predetermined number of uniformly spaced points. This system does not allow interpolating at arbitrary resampling times, and it ignores the effect of the interpolation in the frequency domain.

EP 099, 600 discloses a related system which provides an output signal sampled at an irrational multiple of the input signal sample rate. Again, this reference does not allow interpolating at arbitrary respamling times and ignores the effect of the interpolating operation on the frequency domain.

As can be appreciated from the foregoing discussion, there is a need to provide a method and apparatus for interpolating between data samples such that the interpolated values contain the same frequency information contained in the sampled analog signal, i.e., such that the frequency spectrum of the sampled analog signal is preserved. The present invention is directed to a method and apparatus for interpolating between data samples using a digital filter in a novel manner that achieves this result, i.e., produces interpolated values that preserve the frequency spectrum of an analog signal.

Summary of the Invention

In accordance with this invention, a method and apparatus for producing interpolated values that contain accurate information about the original frequency spectrum of a sampled analog signal is provided, as defined in claims 1 and 5, respectively.

As will be appreciated from the foregoing summary, the present invention provides a method and apparatus for preserving the frequency spectrum of a sampled analog signal by interpolating data samples using a digital filter such that the interpolated values contain frequency domain information that can be used to accurately replicate the frequency spectrum of the analog signal.

Brief Description of the Drawings

The foregoing, and other features and advantages of this invention, will become more readily appreciated as the same becomes further understood by reference to the following detailed description when taken in conjunction with the accompanying drawings, wherein:

FIGURE 1 is a simplified block diagram of a preferred embodiment of an interpolating apparatus formed in accordance with the invention;
FIGURE 2 is a simplified flowchart illustrating the functional steps of a program for controlling the apparatus illustrated in FIGURE 1;
FIGURE 3 is a simplified block diagram depicting an alternative embodiment of the apparatus illustrated in FIGURE 1.
FIGURES 4a, 4b and 4c are more detailed flowcharts of the functional steps illustrated in FIGURE 2;
FIGURE 5 illustrates the relationship between frequency domain information contained in data samples and the frequency response of a suitably designed digital finite impulse response ("FIR") filter suitable for use in the apparatus illustrated in FIGURE 1;
FIGURE 6, lines A-E, illustrates the interrelationship between the frequency response of a digital FIR filter and the continuous impulse response of the digital FIR filter; and,
FIGURE 7, lines A-E, is a series of waveforms illustrating the convolution of the data samples with the continuous impulse response of a four-point digital FIR filter suitable for use in the apparatus illustrated in FIGURE 1.

Description of the Preferred Embodiment

There are situations where it is desirable to analyze, in the frequency domain, data that has been created in the time domain. Such a situation arises when an original analog signal is sampled by a time-based sampling process that produces data samples. While there are numerous interpolation techniques that will provide interpolated values in the time domain based on the data samples, in general, such techniques ignore the frequency domain information contained in the data samples. As a result, the interpolated values may not contain accurate frequency domain information. As a consequence, there has developed a need for an interpolating technique that will produce interpolated values containing accurate frequency domain information. Among other benefits, such values can be used to analyze the frequency spectrum of the original analog signal. Another example of frequency domain information that is of interest to some industries is the changes in phase angle of the shaft of a rotating machine that occur as the machine is operated. The frequency spectrum of analog signals created when such a machine is operated contains shaft phase angle information. As will become obvious from the following discussion, the present invention provides a method and apparatus for preserving the frequency spectrum of an analog signal by interpolating data samples using a digital filter in a novel manner so as to overcome the foregoing problems of the prior art.

FIGURE 1 illustrates a preferred embodiment of an apparatus for interpolating data samples in accordance with the present invention. The apparatus illustrated in FIGURE 1 comprises: a signal processor 10; a sampler 12; a storage device 14; and a clock 16. The sampler 12, the storage device 14 and the clock 16, per se, do not form part of the present invention. They are included so that the invention will be more easily understood. An original fluctuating analog signal, designated V_O, which has an original frequency spectrum, is applied to the sampler 12. The sampler 12 samples the V_O signal and produces a digital data signal comprising data samples, designated V_S, taken at known sample times, designated t_S. The t_S times are provided to the sampler 12 by the clock 16. The V_S values and the t_S times are stored in the storage device 14.

As will be better understood from the following discussion, the signal processor 10 receives control information from an external source, such as time intervals between the t_S times, designated Δt; resample times at which interpolation is to be performed, designated t_R; and, the number of t_R times required, i.e., a data block size, designated Bls. As required, the signal processor 10 also receives the V_S and t_S values stored in the storage device 14. Based upon the externally generated control information and the V_S and t_S values, the signal processor 10 produces a series of interpolated values, designated V_{IV}. The V_{IV} values contain frequency domain information as well as time domain information. As will be better understood from the following discussion, the V_{IV} values can be used to accurately replicate both the time and frequency domain information contained in the V_O signal. The signal processor 10 can take the form of a specialized signal processor, i.e., a signal processor containing a program dedicated to carrying out the invention, or a suitably programmed general purpose computer.

The broad functional steps of the program that controls the signal processor 10 are illustrated in FIGURE 2 and discussed next. Initially, the signal processor 10 reads into its internal memory the V_S values, t_S times, t_R times, Δt intervals, and the data block size, Bls. Next, the program causes the signal processor 10 to read the first t_R time and find a set of V_S values occurring at t_S times that are located near the first t_R time. As will be better understood from the

discussion below, the size of the set of V_S values is determined by the nature of a digital filter, specifically a digital finite impulse response (FIR) filter that, preferably, is implemented in software form in the signal processor 10. The digital FIR filter has a time domain or continuous impulse response function, designated h(x), where x is a normalized time (i.e., x = t/T, where T is defined below and t is some time of interest, such as t_R, for example) and a frequency domain or frequency response function, designated H(y), where y is a normalized frequency (i.e., y = fT). The continuous impulse response function, h(x), has a finite width, T. The size of the set of V_S values is determined by the impulse response width, T. For example, if the impulse response width, T, encompasses four t_S times, the signal processor 10 will find the four V_S values associated with the four t_S times nearest the first resample time, t_R. In this case, the digital FIR filter is commonly referred to as a four-point filter.

After the set of V_S values has been found, the program instructs the signal processor to convolve the continuous impulse response function, h(x), of the digital FIR filter with each of the V_S values in the set, to produce a convolved value, designated V_C, located at t_R, for each of the V_S values in the set. As will become better understood from the following discussion, during this convolution step, the frequency response function, H(y), of the digital FIR filter performs a filtering operation. That is, the filter reduces the effects of spectral images introduced by the sampling process. Next, the signal processor 10 sums the V_C values to produce a first interpolated value, V_{IV}, i.e., the V_{IV} value associated with the first t_R time. Once this V_{IV} value has been computed, the program instructs the signal processor 10 to read the next t_R time, and the functional steps discussed above are repeated. The functional steps illustrated in FIGURE 2 are repeated until the signal processor 10 has produced V_{IV} values for each of the t_R times received and read by the signal processor 10. As a result of the filtering operation discussed above and other aspects of the filtering operation which are discussed below, the V_{IV} values contain accurate frequency spectrum information of the original signal (i.e., V_O).

While the presently preferred embodiment of the invention comprises a signal processor 10 and a program for carrying out the invention, as discussed above and illustrated in FIGURES 1 and 2, it is to be understood that alternative embodiments exist which are also suitable for carrying out the invention. One such alternative embodiment is a hardware embodiment of the sort illustrated in FIGURE 3. In this alternative hardware embodiment, the sampler 12, the storage device 14 and the clock 16 perform the functions discussed above. The alternative apparatus comprises: a receiver 18; a data locator 20; a digital filter 21; a convolver 22; and, a summer 23. The digital filter 21 is preferably a digital FIR filter having the h(x) and H(y) functions noted above.

The receiver 18 receives the V_S values and t_S times from the storage device 14. The data block size, Bls, the time interval between t_S times, Δt, and the resample times, t_R, are provided to the receiver 18 by an external source. The data locator 20 finds the set of t_S times nearest the first t_R time and determines the associated V_S values. As noted above, the size of the V_S value set is determined by the impulse response width, T, of the impulse response function, h(x), of the digital FIR filter 21. Once the V_S values have been found, the convolver 22 convolves the continuous impulse response function, h(x), with the V_S values to produce a V_C value located at the t_R time for each t_S time in the set. Next, the summer 23 sums the V_C values at the t_R time to produce the V_{IV} value associated with the first t_R time. The apparatus illustrated in FIGURE 3 repeats the above process until V_{IV} values have been produced for each t_R time received by the receiver 18. In a manner similar to that discussed above for the signal processor 10 (FIGURE 1), the frequency response function, H(y), of the digital FIR filter 21 performs a filtering operation. As a result, the original frequency spectrum of the V_O signal is accurately reproduced in the frequency domain information contained in the V_{IV} values.

Before returning to the preferred embodiment of the invention (FIGURE 1) and discussing, in more detail, the functional steps of the program illustrated in FIGURE 2, a general "background" discussion of various aspects of the present invention is presented so that the invention may be more easily understood. More specifically, the novel concepts of: using the continuous impulse response function, h(x), of a digital FIR filter to perform time-based interpolation of the data samples, V_S; and, using the frequency response function, H(y), of the digital FIR filter to produce V_{IV} values that contain accurate frequency spectrum information of the original signal (i.e., V_O).

As is well known in the signal sampling art, digitally sampling an analog signal produces spectral images of the analog signal's frequency spectrum. As a result, the data samples produced by a digital sampling process contain frequency domain information relative to both the frequency spectrum of the analog signal and images of the frequency spectrum. When the data samples containing such information are interpolated, a portion of the image information is contributed to the analog signal frequency spectrum information contained in the interpolated values. This contribution of image information results in aliasing errors that distort the frequency spectrum information contained in the interpolated values. As a result, the frequency spectrum information contained in the interpolated values cannot be used to accurately determine the frequency spectrum of the analog signal. As will become better understood from the following discussion, the digital FIR filter used in the present invention reduces the contribution of the spectral image information in the interpolated values. As a result of this and other filtering operations performed by the filter, which are discussed below, the frequency domain information contained in the interpolated values can be used to accurately determine the frequency spectrum of the analog signal.

FIGURE 5 illustrates the relationship between a digital FIR filter suitable for use with the present invention and the frequency spectrum of an analog signal and its spectral images. An original fluctuating analog signal, such as V_O, for example, may be described as having a frequency spectrum 24 with the general shape depicted in FIGURE 5. So as to simplify the following discussion, the frequency spectrum 24 is centered about a zero frequency point. As noted above, spectral images are produced during sampling of the V_O signal. As was also noted above, frequency domain information relative to the frequency spectrum 24 and the spectral images is contained in the resulting data samples, V_S. As is well known in the sampling art, by sampling the V_O signal at a sampling rate, designated f_s, using an oversampling factor, designated m, where m > 1, the spectral images are centered at integer multiples of f_s (i.e., $\pm 1f_s$, $\pm 2f_s$, $\pm 3f_s$, etc.). A representative pair of spectral images 25a and 25b are depicted in FIGURE 5 as dashed curves and are centered at $\pm 1f_s$. As is also well known in the sampling art, the bandwidths of the frequency spectrum 24 and the spectral images 25a and 25b are the same and are a function of the sampling rate, f_s, and the oversampling factor, m. Specifically, the bandwidths are equal to the sampling rate divided by the oversampling factor (i.e., f_s/m). Thus, as illustrated in FIGURE 5, the frequency spectrum 24 has a bandwidth that lies between $\pm f_s/2m$. Likewise, the bandwidth of the representative spectral image 25a lies between $+ f_s(\frac{2m-1}{2m})$ and $+ f_s(\frac{2m+1}{2m})$, and the bandwidth of the other representative spectral image 25b lies between $-f_s(\frac{2m-1}{2m})$ and $-f_s(\frac{2m+1}{2m})$. As noted above, and as will be better understood from the following discussion, a filter, such as a suitably designed digital FIR filter, can reduce the contributions of the spectral images 25a and 25b to the frequency spectrum 24 when the interpolation of the V_S data samples is performed.

In addition to the original frequency spectrum and images 25a and 25b, FIGURE 5 also illustrates an exemplary frequency response 29 for a suitably designed digital FIR filter having the general frequency response function, H(y), discussed above. The frequency response 29 has a shape that is defined by: a main lobe 40; and a series of side lobes 36a and 36b. As is well known in the filtering art, the main lobe 40 comprises a pass band 30 and a pair of transition bands 32a and 32b located on either side of the pass band 30. Transitions between the pass band 30 and the transition bands 32a and 32b are defined by points 62a and 62b, respectively. The series of side lobes 36a and 36b form a pair of stop bands 34a and 34b, located adjacent to transition bands 32a and 32b, respectively. Transitions between the stop band 34a and the transition band 32a, and between the stop band 34b and the transition band 32b are defined by points 60a and 60b, respectively. The pass band 30, transition bands 32a and 32b, and stop bands 34a and 34b are indicated by brackets in FIGURE 5. For ease of illustration, the frequency response 29 is centered at the zero frequency point.

As is well known in the filtering art, the shape of the frequency response 29 is determined, in part, by the oversampling factor, m. More specifically, the relationship between the transition points: 60a and 62a; and, 60b and 62b is fixed and is equal to: 2m - 1; where m > 1. For example, an oversampling factor of two (i.e., m = 2) causes points 60a and 60b to occur at three times the frequency of points 62a and 62b. That is, the stop bands 34a and 34b begin at three times the cutoff frequencies of the pass band 30. In the above example (i.e., m = 2), the pass band 30 lies between $\pm f_s/4$ and the stop bands begin at $\pm 3f_s/4S$ Since the bandwidth of the original signal frequency spectrum 24 is equal to f_s/m, in the above example (i.e., m = 2), the bandwidth is equal to $f_s/2$ and lies between ± fs/4. As a result, the original signal frequency spectrum 24 lies within the pass band 30 of the digital FIR filter and the images 25a and 25b lie within the stop bands 34a and 34b.

If the digital FIR filter were a perfect filter, the stop bands 34a and 34b would be perfectly attenuating, and the frequency information passed by the digital FIR filter would simply be the frequency spectrum 24. However, because real filters are not perfect, the stop bands 34a and 34b have imperfect attenuation capabilities. The series of side lobes 36a and 36b depict the imperfect attenuation of the stop bands 34a and 34b. The peak imperfect attenuation level is illustrated in FIGURE 5 by the horizontal dashed lines 44a and 44b. Because the stop bands 34a and 34b are not perfectly attenuating, portions of the spectral images 25a and 25b that lie within the stop bands 34a and 34b and are below the attenuation levels 44a and 44b are passed by the digital FIR filter. These portions of the images 25a and 25b passed by the digital FIR filter produce aliasing errors in the frequency information passed by the filter. Thus, the passed frequency information cannot be used to accurately replicate the original signal frequency spectrum 24. As will be better understood from the following discussion, the general shape of the frequency response 29 can be altered through appropriate design of the digital FIR filter. For example, the amplitudes of the series of side lobes 36a and 36b can be reduced, thereby increasing the attenuation levels 44a and 44b of the stop bands 34a and 34b. By increasing the stop band attenuation, a smaller portion of the spectral images 25a and 25b will be passed by the filter and the aliasing errors associated with the original signal frequency spectrum 24 passed by the digital FIR filter will be reduced. As a result, original signal frequency spectrum information contained in the interpolated values, V_{IV}, will contain less image information, so that the frequency domain information contained in the V_{IV} values can be used to more accurately replicate the original signal frequency spectrum.

Another source of errors that can distort the original signal frequency spectrum 24 passed by the digital FIR filter is the shape of the pass band 30. In an ideal filter, the pass band 30 would be perfectly flat with respect to frequency. However, in practice, filters are not ideal and the pass band 30 is not perfectly flat. The digital FIR filter illustrated in FIGURE 5 has a pass band ripple 38 that may introduce amplitude errors into the frequency spectrum 24 that is passed

by the filter. As will be better understood from the following discussion, the pass band flatness can be improved through appropriate filter design by reducing the pass band ripple 38. Reducing the pass band ripple 38 reduces the amplitude errors introduced into the frequency spectrum 24 by the ripple. Thus, in accordance with the invention, it is desirable to design a digital FIR filter that has a suitably flat pass band 30 and suitably attenuating stop bands 34a and 34b. An exemplary method for designing such a digital FIR filter suitable for use in the present invention is discussed next.

In the preferred embodiment of the invention, as was noted above, the digital FIR filter is actually implemented in software form within the signal processor 10. As is well known in the filtering art, the frequency response function, H (y), of such a digital FIR filter depicted by the exemplary frequency response 29 in FIGURE 5, can be represented by one or more mathematical equations characterized by poles and zeros in the complex-frequency plane (hereinafter referred to as the frequency plane). As will be better understood from the following discussion, relocating one or more pairs of zeros within the frequency plane is one method of modifying the frequency shape 29. More specifically, the amplitudes of the side lobes 36a and 36b may be adjusted (i.e., reduced) by moving zeros along the real axis in the frequency plane, and the amplitude of the pass band ripple 38 may be adjusted (i.e., reduced) by moving zeros along the imaginary axis of the frequency plane.

As is well known in the filtering art, the frequency response function, H(y), of a filter, such as a digital FIR filter, is related to the continuous impulse response function, h(x), of the filter. Specifically, the h(x) function is the inverse Fourier transform of the H(y) function. Thus, just as with the H(y) function, the h(x) function can be represented by one or more mathematical equations that are the inverse Fourier transform of the equations representing the H(y) function. As a result, changes made to the H(y) function which alter the frequency response 29 will cause corresponding changes in the continuous impulse response function, h(x). As will be presented in the following discussion, each pair of zeros that is moved about the frequency plane adds a cosine term to the equations representing the impulse response function, h(x). Adding cosine terms to the h(x) function changes the shape of the digital FIR filter's impulse response. However, as will be explained more fully below, as long as the number of zeros remains the same within a suitable frequency band, the width of the continuous impulse response, T, remains unchanged.

FIGURE 6 illustratively depicts this relationship between the H(y) and h(x) functions of the digital FIR filter. FIGURE 6, line A illustrates a rectangular continuous impulse response 50. Such a rectangular impulse response 50 is representative of the h(x) function for a perfect FIR filter. The rectangular impulse response 50 has unity height (i.e., h = 1) and width, T. The corresponding H(y) function for a filter having the rectangular impulse response 50 is the Fourier transform of the impulse response 50 and can be defined by an equation of the general form:

$$H(y) = \frac{\sin \pi y}{\pi y}. \qquad (1)$$

The frequency response 29 represented by Equation (1) is illustrated in FIGURE 6 on line B. The zero crossings of the frequency response 29 are the zeros of Equation (1). By normalizing the frequency scale in FIGURE 6 by T, the zeros occur at multiples of 1/T (i.e., y = ±1, ±2, etc.).

The frequency response illustrated in FIGURE 6, line B, can be depicted in a more conventional form by changing the ordinate from a linear to a logarithmic magnitude scale. This causes the negative excursions of the frequency response to become positive excursions while remaining in the same relative position along the frequency axis. The resulting waveform of the frequency response is illustrated in FIGURE 6 on line C and has the same general shape as the frequency response 29 illustrated in FIGURE 5 and discussed above. Accordingly, the frequency response waveform illustrated in FIGURE 6, line C has a main lobe 40 and a series of side lobes 36a and 36b. For reasons of clarity, the pass band 30, transition bands 32a and 32b, and stop bands 34a and 34b are not illustrated in FIGURE 6, however, they have the same characteristics as discussed above and illustrated in FIGURE 5.

FIGURE 6, line D, is a graphical illustration of how relocating zeros along the real axis in the frequency plane can reduce the amplitudes of the side lobes 36a and 36b. For example, by relocating two pairs of zeros that occur at the normalized frequency values of ±1 and ±2 (i.e., y = ±1, ±2), the width of the main lobe 40 is increased. Moving one pair of zeros along the real axis causes a new pair of side lobes 46a and 46b to be inserted where the zero pair has been relocated, such as at y = ±3.5, for example. The reinserted side lobes 46a and 46b reduce the amplitudes of side lobes 42a and 42b, which are adjacent to the main lobe 40. In the above example, the adjacent side lobes 42a and 42b have the highest peak value and therefore determine the attenuation levels 44a and 44b. By moving one pair of zeros from y = ±1, for example, to y = ±3.5, the stop band attenuation levels 44a and 44b are increased to levels 44a' and 44b' and are defined by the reduced amplitudes of the adjacent side lobes 42a and 42b.

A second pair of zeros, originally located at y = ±2 in the above example, can be relocated to a point along the imaginary axis such as y = ± i1.3, for example. The imaginary axis in FIGURE 6, line D is normal to the page and is not shown for reasons of clarity. As noted above, by appropriately positioning the second pair of zeros along the imaginary axis, the pass band ripple 38 (not shown) of the frequency response can be reduced. Thus, by appropriately relocating pairs of zeros associated with the frequency response, the pass band ripple 38 and stop band attenuation

levels 44a and 44b can be improved.

As noted above, when zeros are relocated in the frequency plane, cosine terms are added to the continuous impulse response function, h(x), which change the shape of the h(x) function. In the example illustrated in FIGURE 6, the added cosine terms cause the rectangular impulse response 50 to change to a nonrectangular impulse response 52, whose shape is illustrated in FIGURE 6 on line E. The width, T, of the nonrectangular impulse response 52 remains unchanged because, as was noted above, the total number of zeros in the above example was not changed.

So as to better understand the present invention, a brief discussion of the relationship between the movement of zero pairs in the frequency plane and the addition of cosine terms to the time domain function is presented next. As noted above, Equation (1) represents the frequency response of a perfect filter having the rectangular impulse response 50 illustrated in FIGURE 6, line A. As is well known in the filtering art, a perfect filter, i.e., a rectangular response 50 is not achievable. A more realistic filter has a nonrectangular impulse response (such as the nonrectangular impulse response 52 illustrated in FIGURE 6, line E, for example). Such a nonrectangular impulse response 52 can be represented by introducing a series of cosine terms into Equation (1). The corresponding frequency function, H(y), for such a nonrectangular impulse response 52 can be defined by an equation of the general form:

$$H(y) = \sum_{\ell=-n}^{n} a_\ell \cdot \frac{\sin \pi (y - \ell)}{\pi (y - \ell)} \quad . \qquad (2)$$

where:

n is the number of cosine terms in the time domain;
ℓ is an integer; and,
a_ℓ is coefficient of the ℓ^{th} cosine term.

Equation (2) represents a series of waveforms, each having the general frequency shape defined by Equation (1), centered at different frequencies (i.e., $y - \ell$) and having different amplitudes (i.e., a_ℓ). This is further illustrated by writing Equation (2) in a slightly different form as illustrated by the following equation:

$$H(y) = \sin \pi y \sum_{\ell=-n}^{n} \frac{(-1)^\ell a_\ell}{\pi (y - \ell)} \quad . \qquad (3)$$

The corresponding time domain or continuous impulse response function, h(x), is defined as the inverse Fourier transform of Equation (2), and can be represented by an equation having the general form:

$$h(x) = R(x) \sum_{\ell=0}^{n} (-1)^\ell A_\ell \cos 2\pi\ell x \qquad (4)$$

where:

R(x) is a rectangle having unit width and height.
A_ℓ is a coefficient of a cosine term defined as:

$A_\ell = 2(-1)^\ell a_\ell$ for $\ell \neq 0$; and,
$A_\ell = a_\ell$ for $\ell = 0$; and,

$x = \frac{t}{T}$ where T is the width of the h(x) function.

Thus, as illustrated by Equation (4), a nonrectangular impulse response 52 can be defined by a constant and a series of cosine terms.

As stated above, such a nonrectangular impulse response 52 has the corresponding frequency shape defined by Equation (3). For reasons to be discussed below, Equation (3) can be reduced to a rational fraction having the general

form:

$$H(y) = \frac{y}{\pi} \sin \pi y \, \frac{B_0[(y^2-\alpha_1^2)(y^2-\alpha_2^2)...(y^2-\alpha_n^2)]}{y^2 \times (y^2-1)...(y^2-n^2)} \tag{5}$$

where:

B_0 is a constant; and,
α_ℓ (where ℓ is an integer between 1 and n) is related to A_ℓ in Equation (4). The α_ℓ terms are the roots (i.e., zeros) of the numerator in Equation (5).

As can be seen by comparing Equations (4) and (5), the number of cosine terms in the time domain (Equation (4)) is related to the number of new zeros in the frequency domain (Equation 5)). Moving a zero in the frequency plane is accomplished by adding an appropriate α_ℓ term to Equation (5) that cancels one zero term and replaces the cancelled zero term with a new zero term, thus relocating the cancelled zero to a new location without changing the number of zero terms. Adding α_ℓ terms to Equation (5) so as to relocate a zero causes a cosine term to be added to Equation (4) because, as noted above, the α_ℓ terms are related to the A_ℓ terms (i.e., the cosine coefficients).

Thus, by moving zeros of the H(y) function in the frequency plane so as to improve the stop band attenuation 44a and 44b and improve the flatness of the pass band 30, causes cosine terms to be added to the impulse response in the time domain. As will be readily understood from the foregoing discussion by a person of reasonable skill in this art, the frequency response of a digital FIR filter can be designed so as to optimize the stop band attenuation 44a' and 44b' and pass band ripple 38 of the filter by moving zeros in the frequency plane.

As will be better understood from the following discussion, and in accordance with the preferred embodiment of the present invention, the continuous impulse response function, h(x), can be convolved with the data sample values, V_S, to (eventually) produce the interpolated values, V_{IV}, at the appropriate resample times, t_R. In general, during a convolution operation, the h(x) function is centered at an appropriate sample time, t_S, so that a t_R time is encompassed by the finite impulse response width, T. The h(x) function is multiplied by the V_S value corresponding to the centered t_S time, thereby producing a weighted h(x) function. A convolved value, designated V_C, is the particular value of the weighted h(x) function corresponding to the t_R time encompassed by the impulse response width, T. As will be better understood from the following discussion, a plurality of V_C values for each t_R time are computed when a plurality of weighted h(x) functions overlap the t_R time. The plurality of V_C values are summed at each t_R time to produce the interpolated value, V_{IV}. The convolution steps briefly discussed above are illustrated in FIGURE 7 and discussed in greater detail next by way of an example. In the example which follows, a four-point digital FIR filter is used. As noted above, a four-point filter has a h(x) function whose width, T, encompasses four data samples, (i.e., $4\Delta t \leq T < 5\Delta t$).

FIGURE 7, line A, illustrates an exemplary series of V_S values produced by a time-based sampling process. The V_S values occur at t_S times that, preferably, are spaced apart by equal time intervals, designated Δt. In the above example, each t_S time is assigned an integer value, designated N, starting, for example, with the first t_S time (i.e., N = 1) and increasing sequentially to the last t_S time (i.e., N = Nth value). This is illustrated in FIGURE 7, line A as the integers 1 to N above the series of t_S times. A t_R time, preferably, is located between two adjacent t_S points--t_8 and t_9 or designated integers 8 and 9, for example. In this example, the four t_S times located nearest to t_R, designated integer values of seven through ten (i.e., N = 7-10), are located at sample times, designated t_7 through t_{10}, and have V_S values, designated V_7 through V_{10}, respectively.

The convolution operation is illustrated in FIGURE 7 on lines B-E. The h(x) function is represented by the nonrectangular impulse response shape 52. First, the impulse response 52 is centered at t_7 and is multiplied by the V_7 value to produce a convolution value, designated V_{C1}, located at t_R. Next, the impulse response 52 is centered at t_8 and multiplied by V_8 to produce a convolution value, designated V_{C2}, located at t_R. Similarly, the impulse response 52 is separately located at t_9 and t_{10} and multiplied by V_9 and V_{10} to produce convolution values, designated V_{C3} and V_{C4}, respectively. Next, the V_{C1}-V_{C4} values are summed to produce the V_{IV} value at the t_R time. The convolution process is repeated for subsequent t_R times until a V_{IV} value has been computed for each t_R time.

The convolution operation illustrated in the above example can be expressed in terms of a general convolution equation having the following form:

$$V_{IV} = \sum_{k=1}^{g} V_S(N_k \Delta t)\, h\left(\frac{t_R - N_k \Delta t}{T}\right) \tag{6}$$

where: k is an integer; and, q is equal to the number of t_S times encompassed by the impulse response width, T, (i.e., q = 4 for the four-point digital FIR filter in the above example). The impulse response function, h(x), is expressed in terms of the t_R time offset by an integer multiple of Δt (i.e., $N_k\Delta t$) and therefore forms an offset impulse response function

$$\text{(i.e., } h\left(\frac{t_R - N_k\Delta t}{T}\right)\text{).}$$

Since, as was discussed above, each t_S time is assigned a sequential integer value (i.e., 1 to N), the t_S times located near any t_R time can be defined by the following equation:

$$N_k = N_0 + k \tag{7}$$

where N_0 is the integer multiple of Δt for the t_S time closest to t_R, and k is the integer defined in Equation (6). N_0 can be calculated from the following equation:

$$N_0 = \mathrm{INT}\left(\frac{t_R}{\Delta t} + 0.5\right) \tag{8}$$

where N_0 is rounded off to the nearest integer.

An offset between t_R and N_0, designated t', is computed from the following equation:

$$t' = t_R - N_0\Delta t \tag{9}$$

Solving Equation (7) for N_0 and substituting this value into Equation (8) and rearranging the terms provides the following relationship:

$$t_R - N_k\Delta t = t' - k\Delta t. \tag{10}$$

Substituting the right-hand side of Equation (10) into the impulse response term in Equation (6) yields the following relationship:

$$h\left(\frac{t_R - N_k\Delta t}{T}\right) = h\left(\frac{t' - k\Delta t}{T}\right) \quad . \tag{11}$$

Equation (11) illustrates how the offset impulse response function

$$\text{(i.e., } h\left(\frac{t_R - N_k\Delta t}{T}\right)\text{)}$$

of the digital FIR filter can be expressed in terms of the offset time, t'

$$\text{(i.e., } h\left(\frac{t' - k\Delta t}{T}\right)\text{).}$$

Substituting the right side of Equation (11) into Equation (6) produces the following equation:

$$V_{IV} = \sum_{k=1}^{g} V_S(N_k\Delta t)h\left(\frac{t' - k\Delta t}{T}\right) \tag{12}$$

As illustrated in Equation (12), V_{IV} can be computed from a general convolution equation by multiplying the V_S values (i.e., $V_S(N_k\Delta t)$) by the offset impulse response function

$$\text{(i.e.,}\quad h\left(\frac{t' - k\Delta t}{T}\right)\quad).$$

The products are then summed to produce the V_{IV} value at the resampling time, t_R.

Generally, the value of t' is different for each t_R time and, as a result, the value of

$$h\left(\frac{t' - k\Delta t}{T}\right)$$

must be computed for each t_R time. There are several well-known techniques for computing

$$h\left(\frac{t' - k\Delta t}{T}\right),$$

a few of which are noted next. One technique is to store several precalculated values of the impulse response function (i.e., h(x)) and use these values to compute

$$h\left(\frac{t' - k\Delta t}{T}\right)$$

directly. Another technique is to store only a few precalculated values of h(x) and interpolate to find

$$h\left(\frac{t' - k\Delta t}{T}\right).$$

Additionally, if a large look-up table of cosine terms is available, Equation (4) could be solved directly and the result could be inserted directly into Equation (6). Obviously, several conventional techniques exist for computing the value of

$$h\left(\frac{t' - k\Delta t}{T}\right),$$

and, as such, these techniques do not form a part of the present invention.

As can be seen from examining Equation (12), the present invention provides a representation of a continuous approximation of the V_O signal. That is, for any t_R time desired, Equation (12) can be solved to provide the corresponding V_{IV} value. As noted above, in addition to time domain information, the V_{IV} values computed from Equation (12) contain frequency domain information that can be used to accurately replicate the V_O signal frequency spectrum.

As set forth in the foregoing discussion and in accordance with the preferred embodiment of the present invention, a digital FIR filter is used in a novel manner to interpolate between data samples of an analog signal and produce interpolated values that contain accurate V_O signal frequency spectrum information. As noted above, such a digital FIR filter is preferably implemented in software form in a program that controls a signal processor 10 suitable for carrying out the invention. The broad functional steps of such a program are illustrated in FIGURE 2 and discussed above. A more detailed flow chart of the program depicted in FIGURE 2 is illustrated in FIGURES 4a, 4b, and 4c and discussed next. The steps set forth in FIGURES 4a-4c are examples of a four-point digital FIR filter application, such

as for the four-point filter discussed in the above example. Obviously, the steps depicted in FIGURES 4a-4c can be readily adapted to other digital FIR filters (such as a ten-point filter, for example).

Initially, as illustrated in FIGURE 4a, the program instructs the signal processor 10 to read and store: the V_S values; the t_S times; the t_R times; the Δt interval; and, the data block size, Bls. The program further instructs the signal processor 10 to open a temporary file, $V_{IV}(I)$, for storing the computed V_{IV} values.

Next, the program causes the signal processor 10 to assign sequential integer values, N, to the t_S times that have been read and stored. Preferably, the signal processor 10 begins with the first t_S time and proceeds sequentially to the last t_S time. That is, for example, if there are ten t_S times, they are consecutively numbered 1-10 from the first to the last t_S time. Next, the signal processor 10 is instructed to read the first t_R time.

The next functional step performed by the signal processor 10 is to determine the set of t_S times that are closest to the first t_R time. As noted above, the size of the set is determined by the width of the impulse response, T. First, a resampling index counter, designated I, and a summation index counter, designated K, are initialized (i.e., I = 1 and K = 1). Next, the program instructs the signal processor 10 to determine the closest t_S time to the first t_R time, i.e., N_0. N_0 is computed from the following equation:

$$N_0 = \mathrm{INT}\left[\left(\frac{Bls \cdot t_R}{\Delta t}\right) + 0.5 + 1\right] \quad . \tag{13}$$

Next, the offset, t', between the first resample time, t_R, and the nearest t_S time, (i.e., $N_0 \cdot \Delta t$) is computed from Equation (9) above. Once N_0 and t' have been computed, the program instructs the signal processor 10 to find the other t_S times, in the set of t_S times closest to t_R. In the present example, i.e., a four-point digital FIR filter application, the signal processor 10 will locate the four t_S times closest to t_R. Specifically, and as illustrated in FIGURE 4b, if t' is equal to or greater than half the interval between adjacent t_S times (i.e., $t' \geq \Delta t/2$), then the four closest t_S times are determined by the following equation:

$$N_K = N_0 + K - C \quad . \tag{14}$$

If t' is less than half the Δt interval (i.e., $t' < \Delta t/2$), then the four closest t_S values are determined by the following equation:

$$N_K = N_0 + C - K \quad . \tag{15}$$

In Equations (14) and (15), K is the summation index counter value noted above and C is an integer whose value is one-half the number of t_S times encompassed by T (i.e., where C = 2 for the four-point filter in the above example).

After the first N_K point has been determined from Equation (14) or (15), the program determines whether the N_K point lies within the data block (i.e., $1 \leq N_K \leq Bl_s$) and, if not, adjusts the N_K value so that it is within the data block. More specifically, if the integer value of the t_S time lies outside the data block (i.e., N_K > Bls), and is greater than or equal to one (i.e., $N_K \geq 1$), the program places the t_S time within the data block by subtracting the data block size from the integer value of the t_S time (i.e., $N_K = N_K - Bls$). The signal processor 10 stores this value of N_K. If the integer value of the t_S time is outside the data block and is less than one (i.e., $N_K < 1$), then the program adds the data block size to the integer value of the t_S time (i.e., $N_K = N_K + Bls$) and stores this value of N_K. If the integer value of the t_S time is within the data block (i.e., $1 \leq N_K \leq Bl_s$), the value of N_K is stored.

Next, the program determines if all of the t_S times in the set have been found. Specifically, in the four-point filter example discussed above, if the summation index counter value is less than four (i.e., K < 4) then the counter is incremented (i.e., K = K + 1) and the above process is repeated. A summation index counter value greater than or equal to four (i.e., K ≥ 4) indicates that the four t_S times in the set have been found and the program proceeds to the next functional step, which is described next and illustrated in FIGURE 4c.

Once the appropriate number of t_S times near the t_R time have been found, the signal processor 10 convolves the continuous impulse response function (h(x)), of a suitably designed digital FIR filter with the V_S values of the t_S times found in the set discussed above. As noted above, a suitable digital FIR filter is preferably implemented in software form and can be characterized by one or more mathematical formulae, such as Equation (2) (i.e., the frequency response function, H(y)) and Equation (4) (i.e., the impulse response function, h(x)), for example. The program instructs

the signal processor 10 to determine a particular value of the offset impulse response function,

$$h\left(\frac{t' - K\Delta t}{T}\right),$$

defined in Equation (11) above, where k is replaced by K in the program illustrated in FIGURE 4C. As discussed above, the value of

$$h\left(\frac{t' - K\Delta t}{T}\right)$$

can be determined by conventional techniques. Next, the signal processor 10 performs the convolution operation by using the value of

$$h\left(\frac{t' - K\Delta t}{T}\right)$$

in the general convolution equation set forth in Equation (12) where, as noted above, k = K and q = 4. The resulting V_{IV} value is stored as a V_{IV} (I) value. This last step in the program performs the two functional steps illustrated in FIGURE 2 and discussed above. That is, the functional steps of producing the V_C values and summing them to produce a V_{IV} value are performed by the signal processor 10 in the convolution operation step illustrated in FIGURE 4C.

Once the V_{IV} value has been computed and stored by the signal processor 10, the program increments the resampling counter, I (i.e., I = I + 1). If the resampling counter value is less than or equal to the total number of resample times read by the signal processor 10 (i.e., I ≤ BIs), then the next t_R time is read and the above steps are repeated. A resampling counter value that exceeds the total number of resample times (i.e., I > BIs) indicates that the V_{IV} values have been computed for all t_R times read by the signal processor 10 and the program is exited.

As can be readily appreciated from the foregoing description, the invention provides a method and apparatus for preserving the frequency spectrum of a sampled analog signal using a digital FIR filter in a novel manner for interpolating between data samples. As a result, the interpolated values contain frequency information that can be used to accurately replicate the frequency spectrum of the analog signal. While a preferred embodiment of the invention has been illustrated and described herein, it is to be understood that, within the scope of the appended claims, various changes can be made. For example, digital FIR filters other than the four-point filter discussed in the above example can also be used. In fact, even non-FIR filters (i.e., filters having continuous impulse responses of infinite width) can be utilized, however, the convolution operation becomes much more complex. The frequency response and impulse response of the digital FIR filter can also be different than as illustrated and discussed above. Likewise, an oversampling factor other than two (i.e., 1 < m = 2) could also be used. While the time interval, Δt, between adjacent data samples is preferably the same, the invention is capable of performing its intended function even when the data samples are produced by a sampling process that samples the analog signal at a variable rate. Furthermore, the program merely illustrates one possible method of carrying out the invention and it is to be understood that other equally suitable programs could be utilized.

Claims

1. A method of producing interpolated values of an analog signal which has been sampled at known sample times (ts), the method comprising the steps of:

   (a) storing data samples (Vs) of the analog signal, said data samples containing frequency domain information about the analog signal's original frequency spectrum and frequency domain information about images of the analog signal's original frequency spectrum;
   (b) receiving an arbitrary selection of desired resample times (tr) at which said data samples are to be interpolated; and for each resample time (tr) :
   (c) choosing a set of data samples (Vs) occuring at sample times (ts) that are located nearest the resample time (tr);
   (d) weighting said set of data samples (Vs) by convoluting each of said data samples (Vs) with a continuous

nonrectangular impulse response function h(x) of a filter having a predetermined frequency response H(y) comprising a substantially flat passband and suitably attenuating stopbands, said weighting producing a plurality of convolution values at each of said arbitrary selection of resample times (tr); and,
(e) summing said plurality of convolution values at each of said resample times and producing an interpolated value at each of said resample times (tr).

2. Method according to claim 1, wherein the size of the set of sample values (Vs) is determined by the width of the impulse response of the filter.

3. The method according to one of the preceding claims, wherein the analog signal is sampled at a sampling frequency, fs, with an oversampling factor, m, and the predetermined frequency response includes a substantially flat passband for a band of frequencies between - fs/2m and fs/2m, and suitably attenuating stopbands for frequencies above fs $\frac{(2m-1)}{2m}$ and below -fs $\frac{(2b-1)}{2m}$.

4. Method according to one of claims 1 to 3, wherein a frequency function for the continuous nonrectangular impulse response has a general form as follows:

$$H(y) = \sum_{L=-n}^{n} a_L \cdot \frac{\sin \pi(y - L)}{\pi(y - L)} .$$

5. An apparatus for producing interpolated values of an analog signal which has been sampled at known sampling times (ts), the apparatus comprising:

(a) a receiver (18) for receiving data samples (Vs) of the analog signal, said data samples contain frequency domain information about the analog signal's original frequency spectrum and frequency domain information about images of the analog signal's original frequency spectrum, and for receiving an arbitrary selection of resample times (tr) at which said data samples (Vs) are to be interpolated; and,
(b) interpolating means (20, 21, 22, 23) coupled to said receiver (18) for receiving said data samples (Vs) and said selection of resample times (tr), said interpolating means interpolating between said data samples to produce interpolated values (Viv) at said resample times (tr), wherein said interpolating means includes

(b1) a filter (21) producing a continuous nonrectangular impulse response h(x) and an associated frequency response (Hy) comprising a substantially flat pass-band and suitably attenuating stopbands;
(b2) a data locator (20) coupled to said receiver (18) for receiving said data samples (Vs) and said resample times (tr) and locating a set of said data samples (Vs) for each resample time (tr) that is located nearest each of said resample times (tr);
(b3) convolving means (22), coupled to said data filter (21) and said data locator (20), for weighting the set of the data samples (Vs) with the continuous nonrectangular impulse response of the filter (21) for each of said resample times (tr) to produce a plurality of convolution values at the respective resample times (tr); and
(b4) a summer (23) coupled to the convolving means (22) which sums the plurality of convolution values at each of the resample times to produce one of the interpolated values (Viv) at each of the resample times (tr).

6. The apparatus according to claim 5 wherein a frequency function for the continuous nonrectangular impulse response has a general form as follows:

$$H(y) = \sum_{L=-n}^{n} a_L \cdot \frac{\sin \pi(y - L)}{\pi(y - L)} .$$

7. The apparatus according to claim 5 or 6, wherein said frequency response of said filter (21) comprises:

(a) a pass band for passing frequency domain information about the frequency spectrum of the analog signal and reducing contribution from pass band errors contained in said interpolated values; and,
(b) stop bands for reducing contribution from images of the frequency spectrum of the analog signal contained

in said interpolated values.

8. The apparatus according to one of claims 5 to 7, wherein said filter (21) is a digital filter.

9. The apparatus according to one of claims 5 to 8, wherein said filter (21) is a digital finite impulse response filter.

10. The apparatus according to one of claims 5 to 9, wherein said filter (21) is implemented in software form.

Patentansprüche

1. Verfahren zum Erzeugen interpolierter Werte eines analogen Signals, das bei bekannten Abtastzeiten (ts) abgetastet wurde, mit folgenden Verfahrensschritten:

   (a) Speichern der Datenabtastwerte (Vs) des analogen Signals, wobei die Datenabtastwerte Frequenzbereichsinformation über das ursprüngliche Frequenzspektrum des analogen Signals und Frequenzbereichsinformation über Bilder des ursprünglichen Frequenzspektrums des analogen Signals enthalten;
   (b) Empfangen einer willkürlichen Auswahl erwünschter erneuter Abtastzeiten (tr), zu denen die Datenabtastwerte interpoliert werden sollen; und
   für jede erneute Abtastzeit (tr):
   (c) Wählen einer Gruppe aus Datenabtastwerten (Vs), welche bei den Abtastzeiten (ts) auftreten, die am nächsten bei der erneuten Abtastzeit liegen;
   (d) Gewichten der Gruppe der Datenabtastwerte (Vs) durch Falten jedes Datenabtastwertes (Vs) mit einer kontinuierlichen nicht rechteckigen Impulsantwortfunktion h(x) eines Filters mit einem vorgegebenen Frequenzgang H(y), welcher einen im wesentlichen flachen Durchlaßbereich und geeignet dämpfende Sperrbereiche aufweist, wobei das Gewichten mehrerer Faltungswerte bei jeder der willkürlich ausgewählten erneuten Abtastzeiten (tr) erzeugt; und
   (e) Summieren der mehreren Faltungswerte bei jeder der erneuten Abtastzeiten und Erzeugen eines interpolierten Wertes bei jeder der erneuten Abtastzeiten (tr).

2. Verfahren nach Anspruch 1, bei dem die Größe der Gruppen der Abtastwerte (Vs) durch die Breite der Impulsantwort des Filters bestimmt wird.

3. Verfahren nach einem der vorangehenden Ansprüche, bei dem das analoge Signal mit einer Abtastfrequenz, fs, mit einem Überabtastungsfaktor, m, abgetastet wird, und die vorgegebene Frequenzantwort einen im wesentlichen flachen Durchlaßbereich für ein Frequenzband zwischen -fs/2m und fs/2m sowie geeignet dämpfende Sperrbereiche für Frequenzen über fs $\frac{(2m-1)}{2m}$ und unter -fs $\frac{(2m-1)}{2m}$ aufweist.

4. Verfahren nach einem der Ansprüche 1 bis 3, bei dem eine Frequenzfunktion für die kontinuierliche nicht rechteckige Impulsantwort die folgende allgemeine Form hat:

$$H(y) = \sum_{L=-n}^{n} a_L \cdot \frac{\sin\pi(y-L)}{\pi(y-L)} .$$

5. Vorrichtung zum Erzeugen interpolierter Werte eines analogen Signals, welches bei bekannten Abtastzeiten (ts) abgetastet wurde, mit folgenden Merkmalen:

   (a) einem Empfänger (18) zum Empfangen von Datenabtastwerten (Vs) des analogen Signals, wobei die Datenabtastwerte Frequenzbereichsinformation über das ursprüngliche Frequenzspektrum des analogen Signals und Frequenzbereichsinformation über Bilder des ursprünglichen Frequenzspektrums des analogen Signals enthalten, und zum Empfangen einer willkürlichen Auswahl erneuter Abtastzeiten (tr), bei welchen die Datenabtastwerte (vs) interpoliert werden sollen;
   (b) Interpolationsmittel (20, 21, 22, 23), welche mit dem Empfänger (18) verbunden sind, zum Empfangen der Datenabtastwerte (Vs) und der erneuten Abtastzeiten (tr) wobei die Interpolationsmittel zwischen den Datenabtastwerten interpolieren, um interpolierte Werte (Viv) bei den erneuten Abtastzeiten (tr) zu erzeugen, wobei die Interpolationsmittel folgende Merkmale aufweisen:

(b1) ein Filter (21), welches eine kontinuierliche nicht rechteckige Impulsantwort h(x) und einen zugehörigen Frequenzgang H(y) mit einem im wesentlichen flachen Durchlaßbereich und geeignet dämpfenden Sperrbereichen erzeugt;
(b2) einen Datenlokalisierer (20), der mit dem Empfänger (18) verbunden ist, zum Empfang der Datenabtastwerte (Vs) und der erneuten Abtastzeiten (tr) und zum Lokalisieren einer Gruppe aus Datenabtastwerten (Vs) für jede erneute Abtastzeit (tr), welche am nächsten bei jeder der erneuten Abtastzeiten (tr) liegt,
(b3) Faltungsmittel (22), welche mit dem Datenfilter (21) und dem Datenlokalisierer (20) verbunden sind, zum Gewichten der Gruppe der Datenabtastwerte (Vs) mit der kontinuierlichen nicht rechteckigen Impulsantwort des Filters (21) für jede der erneuten Abtastzeiten (tr), um mehrere Faltungswerte bei den jeweiligen erneuten Abtastzeiten (tr) zu erzeugen,
(b4) einen Summierer (28), der mit den Faltungsmitteln (22) verbunden ist und die mehreren Faltungswerte bei jeder der erneuten Abtastzeiten summiert, um bei jeder der erneuten Abtastzeiten (tr) einen der interpolierten Werte (Viv) zu erzeugen.

6. Vorrichtung nach Anspruch 5, bei der die Frequenzfunktion für die kontinuierliche nicht rechteckige Impulsantwort die folgende allgemeine Form hat:

$$H(y) = \sum_{L=-n}^{n} a_L \cdot \frac{\sin \pi(y - L)}{\pi(y - L)} .$$

7. Vorrichtung nach Anspruch 5 oder 6, bei der der Frequenzgang des Filters (21) folgende Merkmale aufweist:

(a) einen Durchlaßbereich zum Durchlassen von Frequenzbereichsinformationen über das Frequenzspektrum des analogen Signals und zum Reduzieren von Beiträgen aus Durchlaßbereichsfehlern, welche in den interpolierten Werten enthalten sind; und
(b) Sperrbereiche zum Reduzieren von Beiträgen aus Bildern des Frequenzspektrums des analogen Signals, welche in den interpolierten Werten enthalten sind.

8. Vorrichtung nach einem der Ansprüche 5 bis 7, bei der das Filter (21) ein digitales Filter ist.

9. Vorrichtung nach einem der Ansprüche 5 bis 8, bei der das Filter (21) ein digitales FIR-Filter ist.

10. Vorrichtung nach einem der Ansprüche 5 bis 9, bei der das Filter (21) in Softwareform realisiert ist.

Revendications

1. Procédé pour produire des valeurs interpolées d'un signal analogique qui a été échantillonné à des temps d'échantillonnage connus (ts), le procédé comprenant les étapes consistant à:

(a) mémoriser des échantillons de données (Vs) du signal analogique, lesdits échantillons de données contenant une information de domaine fréquentiel concernant le spectre de fréquence d'origine du signal analogique et une information de domaine fréquentiel concernant des images du spectre de fréquence d'origine du signal analogique;
(b) recevoir une sélection arbitraire de temps de rééchantillonnage souhaités (tr) auxquels lesdits échantillons de données doivent être interpolés; et
pour chaque temps de rééchantillonnage (tr):
(c) choisir un ensemble d'échantillons de données (Vs) apparaissant à des temps d'échantillonnage (ts) qui sont situés les plus proches du temps de rééchantillonnage (tr);
(d) pondérer ledit ensemble d'échantillons de données (Vs) en convoluant chacun desdits échantillons de données (Vs) avec une fonction de réponse impulsionnelle non rectangulaire continue h(x) d'un filtre ayant une réponse en fréquence prédéterminée H(y) comprenant une bande passante sensiblement plate et des bandes d'arrêt avec une atténuation appropriée, ladite pondération produisant une pluralité de valeurs de convolution à chacun des temps de ladite sélection arbitraire de temps de rééchantillonnage (tr); et
(e) additionner ladite pluralité de valeurs de convolution à chacun desdits temps de rééchantillonnage et pro-

duire une valeur interpolée à chacun desdits temps de rééchantillonnage (tr).

2. Procédé selon la revendication 1, dans lequel la taille de l'ensemble de valeurs d'échantillons (Vs) est déterminée par la largeur de la réponse impulsionnelle du filtre.

3. Procédé selon l'une quelconque des revendications précédentes, dans lequel le signal analogique est échantillonné à une fréquence d'échantillonnage fs avec un facteur de suréchantillonnage m, et la réponse en fréquence prédéterminée inclut une bande passante sensiblement plate pour une bande de fréquence entre $-f_s/2m$ et $f_s/2m$, et des bandes d'arrêt avec une atténuation appropriée pour les fréquences supérieures à $f_s(2m-1)/2m$ et inférieures à $-f_s(2b-1)/2m$.

4. Procédé selon l'une quelconques des revendications 1 à 3, dans lequel une fonction de fréquence pour la réponse impulsionnelle non rectangulaire continue a la forme générale suivante:

$$H(y) = \sum_{L=-n}^{n} \alpha_L[\sin\pi(y-L)]/[\pi(y-L)].$$

5. Dispositif pour produire des valeurs interpolées d'un signal analogique qui a été échantillonné à des temps d'échantillonnage connus (ts), le dispositif comprenant:

(a) un récepteur (18) pour recevoir des échantillons de données (Vs) du signal analogique, lesdits échantillons de données contenant une information de domaine fréquentiel concernant le spectre de fréquence d'origine du signal analogique et une information de domaine fréquentiel concernant des images du spectre de fréquence d'origine du signal analogique, et pour recevoir une sélection arbitraire de temps de rééchantillonnage (tr) auxquels lesdits échantillons de données (Vs) doivent être interpolés; et
(b) des moyens d'interpolation (20, 21, 22, 23) couplés audit récepteur (18) pour recevoir lesdits échantillons de données (Vs) et ladite sélection de temps de rééchantillonnage (tr), lesdits moyens d'interpolation réalisant une interpolation entre lesdits échantillons de données pour produire des valeurs interpolées (V_{IV}) auxdits temps de rééchantillonnage (tr), dans lequel lesdits moyens d'interpolation incluent

(b1) un filtre (21) produisant une réponse impulsionnelle non rectangulaire continue h(x) et une réponse en fréquence (Hy) associée comprenant une bande passante sensiblement plate et des bandes d'arrêt avec une atténuation appropriée;
(b2) un circuit de localisation de données (20) couplé audit récepteur (18) pour recevoir lesdits échantillons de données (Vs) et lesdits temps de rééchantillonnage (tr) et localiser un ensemble desdits échantillons de données (Vs) pour chaque temps de rééchantillonnage (tr) qui est situé le plus proche de chacun desdits temps de rééchantillonnage (tr);
(b3) des moyens de convolution (22), couplés audit filtre de données (21) et audit circuit de localisation de données (20), pour pondérer l'ensemble des échantillons de données (Vs) avec la réponse impulsionnelle non rectangulaire continue du filtre (21) pour chacun desdits temps de rééchantillonnage (tr) afin de produire une pluralité de valeurs de convolution aux temps de rééchantillonnage (tr) respectifs; et
(b4) un additionneur (23) couplé aux moyens de convolution (22) qui additionnent la pluralité de valeurs de convolution, à chacun des temps de rééchantillonnage, pour produire une des valeurs interpolées (V_{IV}) à chacun des temps de rééchantillonnage (tr).

6. Dispositif selon la revendication 5, dans lequel une fonction de fréquence pour la réponse impulsionnelle non rectangulaire continue a la forme générale suivante:

$$H(y) = \sum_{L=-n}^{n} \alpha_L[\sin\pi(y-L)]/[\pi(y-L)].$$

7. Dispositif selon la revendication 5 ou 6, dans lequel ladite réponse en fréquence dudit filtre (21) comprend:

(a) une bande passante pour transmettre une information de domaine fréquentiel concernant le spectre de fréquence du signal analogique et réduire une contribution provenant d'erreurs de bande passante contenues dans lesdites valeurs interpolées; et
(b) des bandes d'arrêt pour réduire une contribution provenant d'images du spectre de fréquence du signal analogique contenues dans lesdites valeurs interpolées.

8. Dispositif selon l'une quelconque des revendications 5 à 7, dans lequel ledit filtre (21) est un filtre numérique.

9. Dispositif selon l'une quelconque des revendications 5 à 8, dans lequel ledit filtre (21) est un filtre numérique à réponse impulsionnelle finie.

10. Dispositif selon l'une quelconque des revendications 5 à 9, dans lequel ledit filtre (21) est implanté sous la forme d'un logiciel.

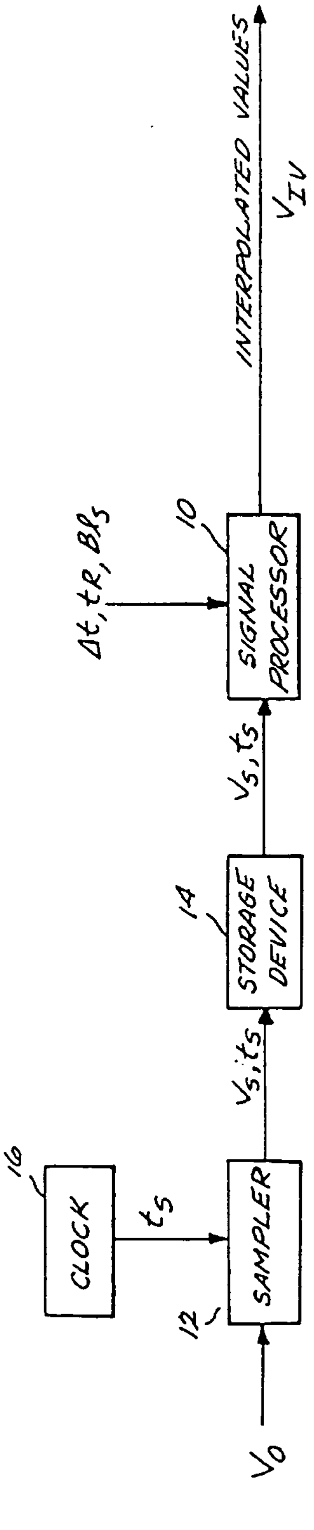

CLOCK
10
t_S
SAMPLER
12
V_O
V_S, t_S
STORAGE DEVICE
14
V_S, t_S
SIGNAL PROCESSOR
10
$\Delta t, t_R, Bl_S$
INTERPOLATED VALUES
V_{IV}

Fig. 1.

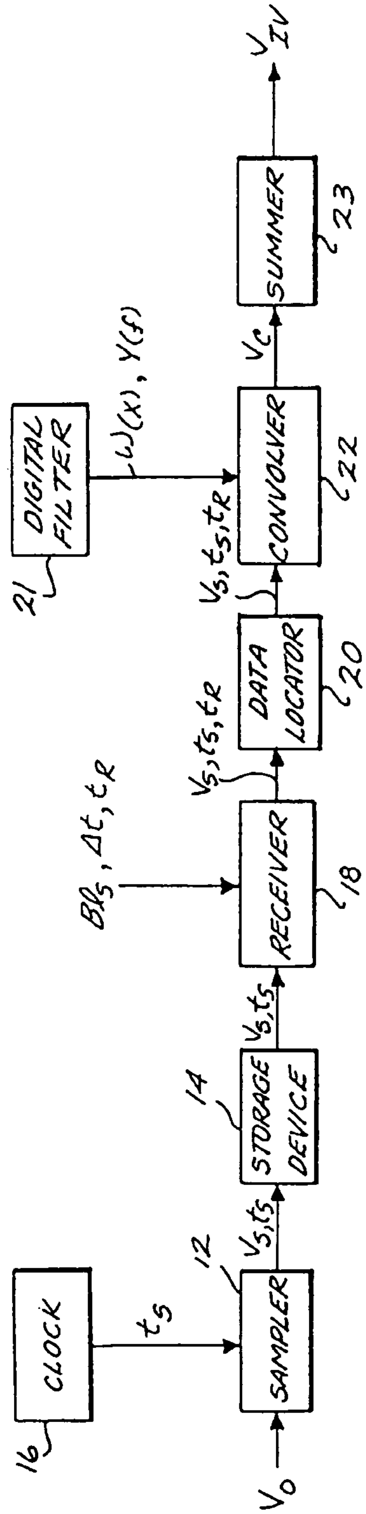

CLOCK
16
t_S
SAMPLER
12
V_O
V_S, t_S
STORAGE DEVICE
14
V_S, t_S
RECEIVER
18
$Bl_S, \Delta t, t_R$
V_S, t_S, t_R
DATA LOCATOR
20
V_S, t_S, t_R
DIGITAL FILTER
21
$\omega(x), Y(f)$
CONVOLVER
22
V_C
SUMMER
23
V_{IV}

Fig. 3.

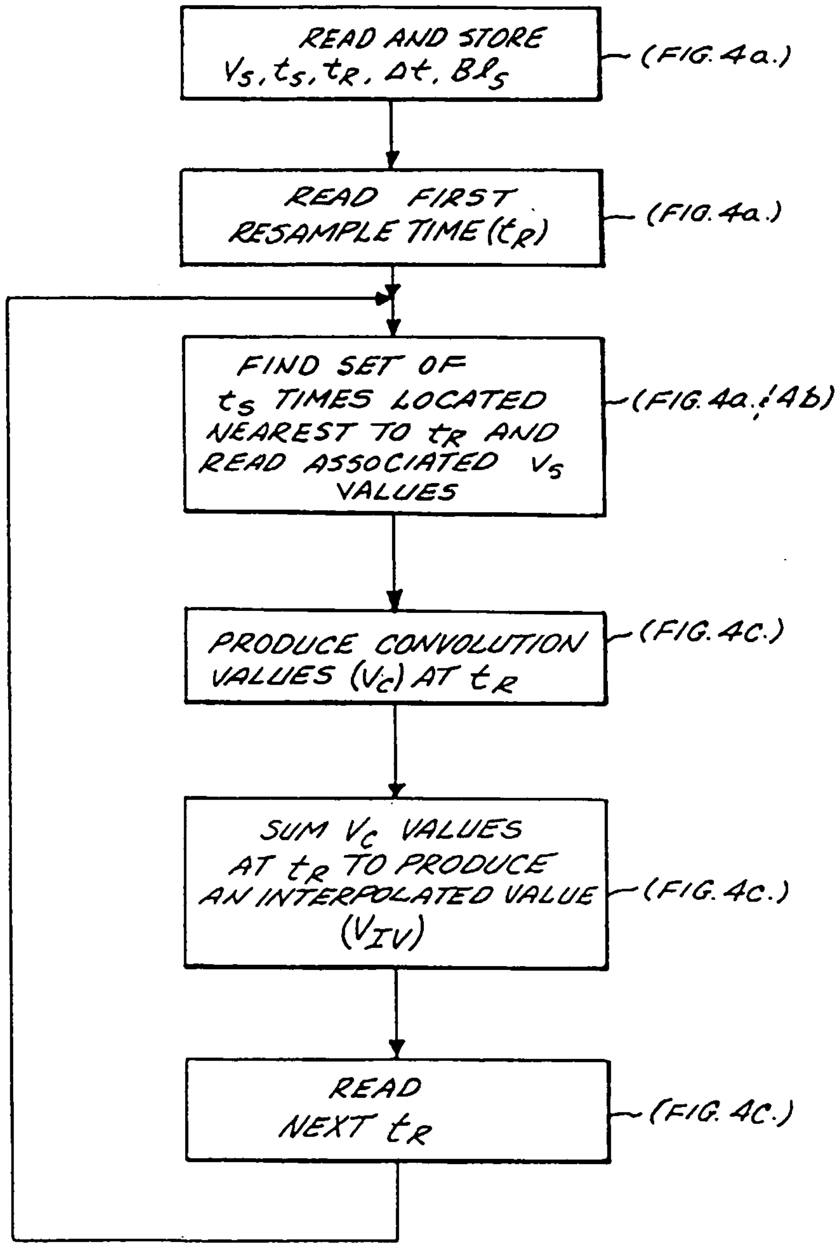

READ AND STORE
$V_S, t_S, t_R, \Delta t, Bl_S$
(FIG. 4a.)
READ FIRST
RESAMPLE TIME (t_R)
(FIG. 4a.)
FIND SET OF
t_S TIMES LOCATED
NEAREST TO t_R AND
READ ASSOCIATED V_S
VALUES
(FIG. 4a. & 4b)
PRODUCE CONVOLUTION
VALUES (V_C) AT t_R
(FIG. 4c.)
SUM V_C VALUES
AT t_R TO PRODUCE
AN INTERPOLATED VALUE
(V_{IV})
(FIG. 4c.)
READ
NEXT t_R
(FIG. 4C.)

Fig. 2.

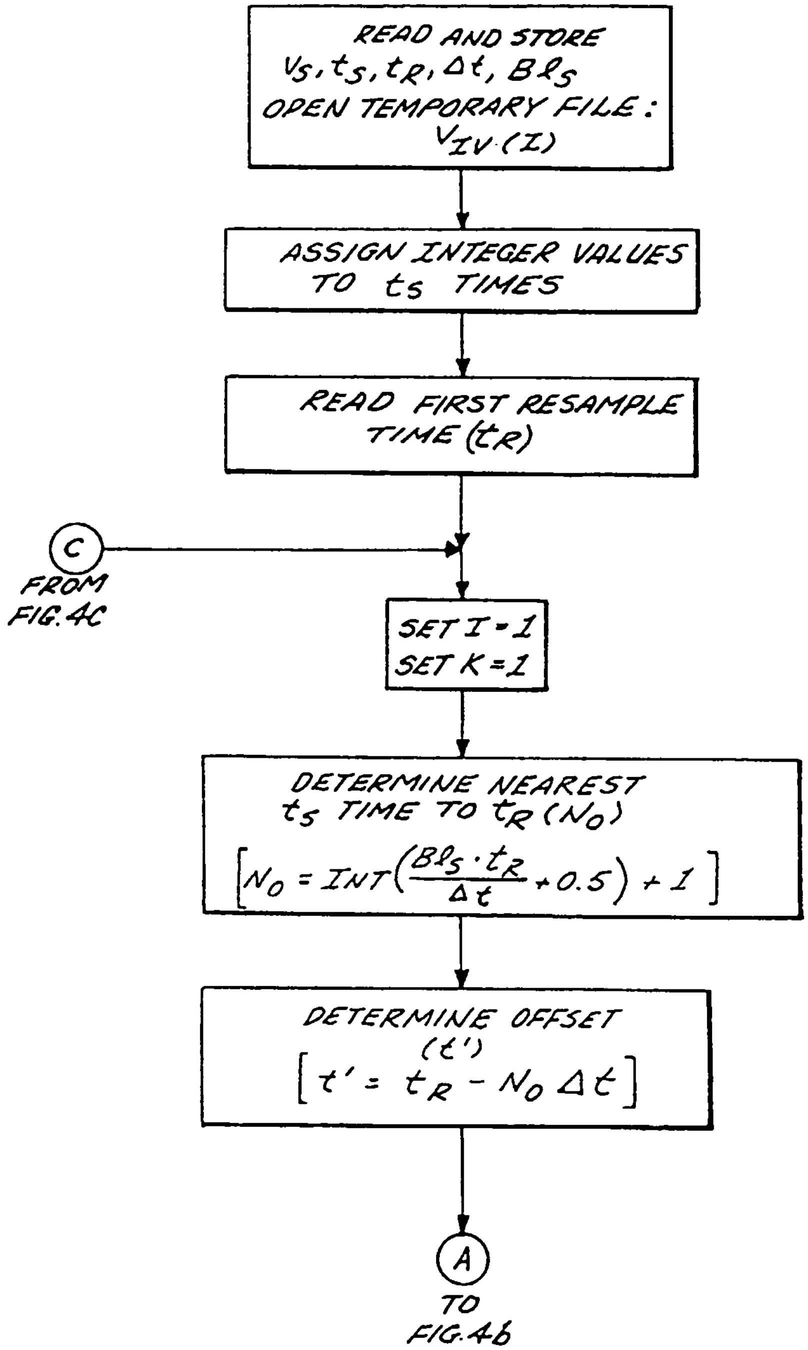

READ AND STORE
$V_S, t_S, t_R, \Delta t, Bl_S$
OPEN TEMPORARY FILE :
$V_{IV}(I)$
ASSIGN INTEGER VALUES TO t_S TIMES
READ FIRST RESAMPLE TIME (t_R)
C
FROM FIG. 4C
SET I = 1
SET K = 1
DETERMINE NEAREST t_S TIME TO t_R (N_0)
$\left[N_0 = INT\left(\frac{Bl_S \cdot t_R}{\Delta t} + 0.5\right) + 1\right]$
DETERMINE OFFSET (t')
$\left[t' = t_R - N_0 \Delta t\right]$
A
TO FIG. 4b

Fig. 4a.

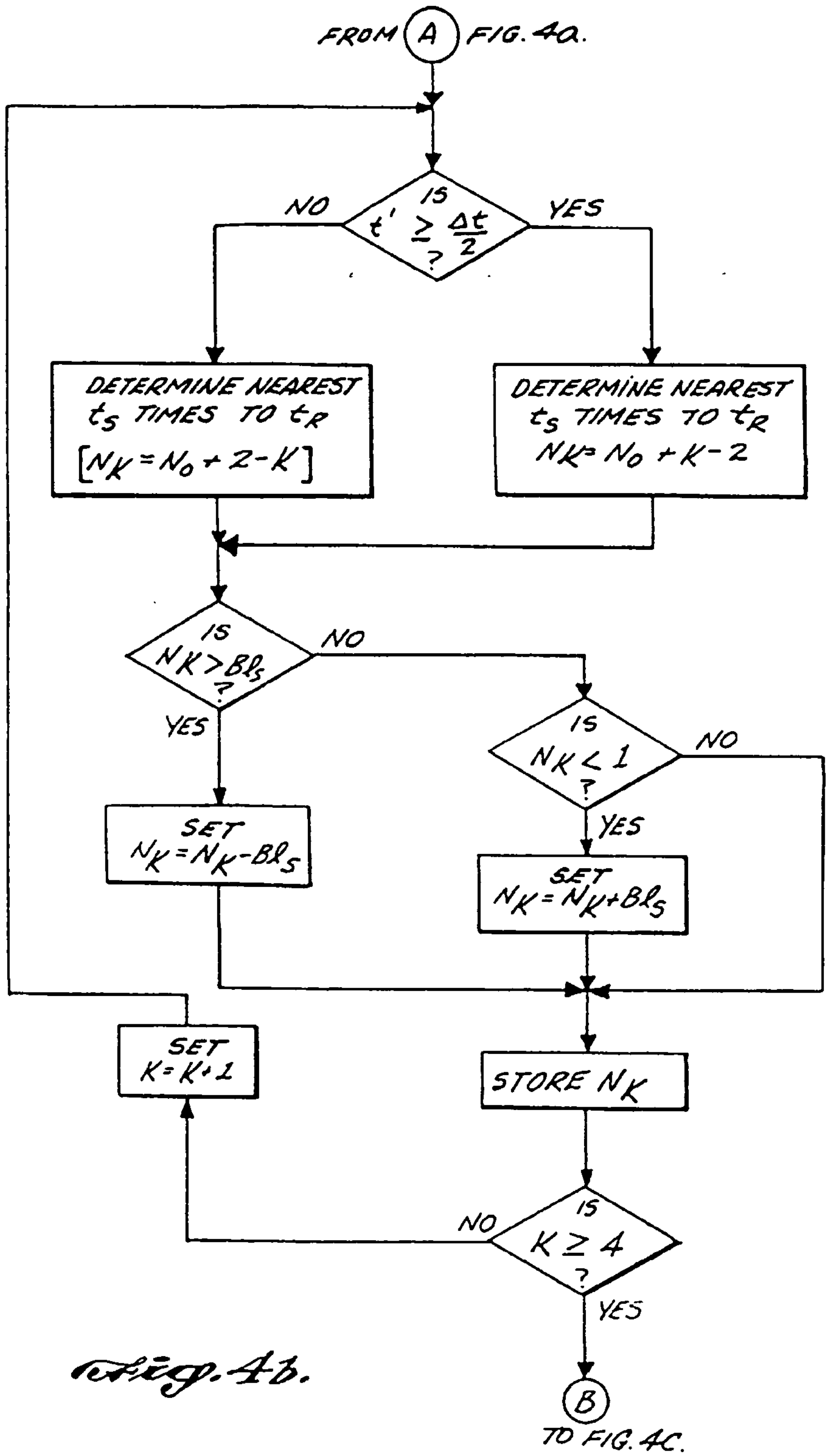

FROM A FIG. 4a.
IS t' ≥ Δt/2 ?
NO
YES
DETERMINE NEAREST t_S TIMES TO t_R [$N_K = N_0 + 2 - K$]
DETERMINE NEAREST t_S TIMES TO t_R $N_K = N_0 + K - 2$
IS $N_K > Bl_S$?
NO
YES
IS $N_K < 1$?
NO
YES
SET $N_K = N_K - Bl_S$
SET $N_K = N_K + Bl_S$
SET $K = K + 1$
STORE N_K
IS $K \geq 4$?
NO
YES
B
TO FIG. 4C.

Fig. 4b.

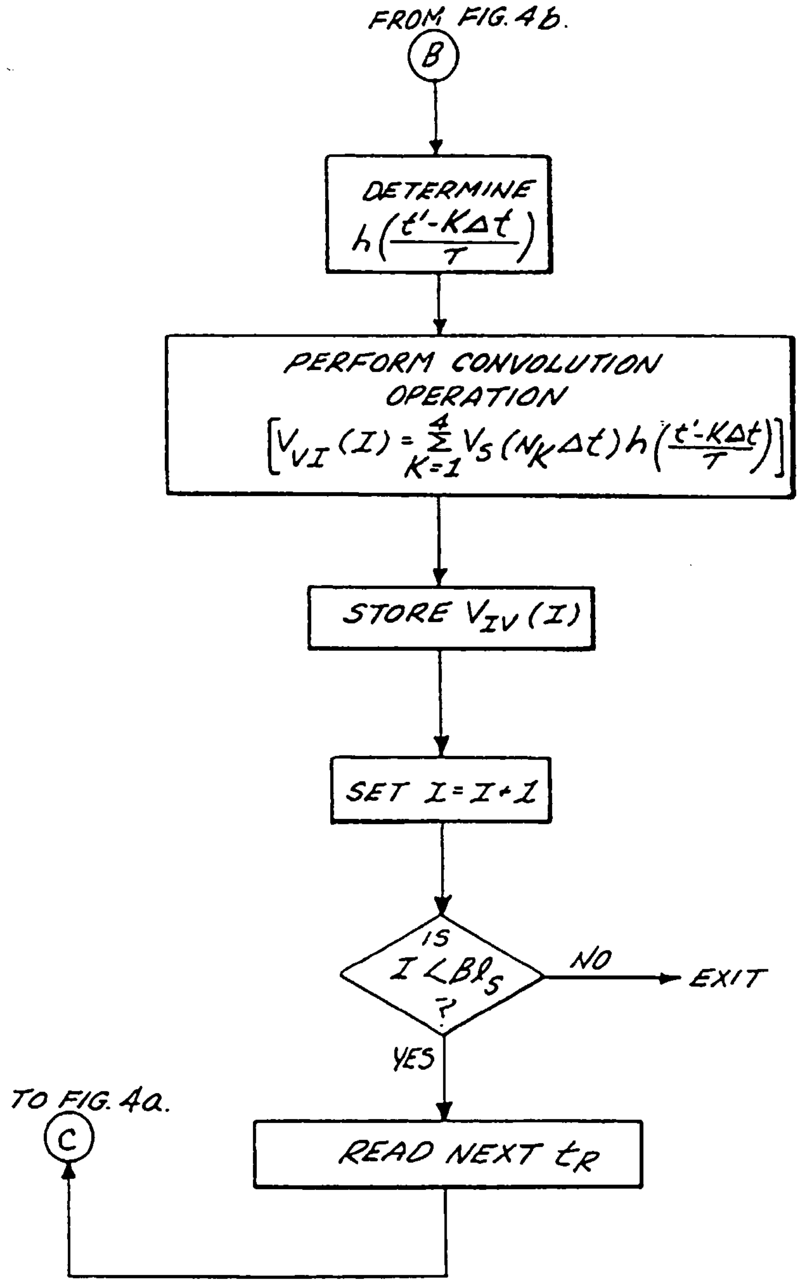

FROM FIG. 4b.
B
DETERMINE
$h\left(\frac{t'-K\Delta t}{T}\right)$
PERFORM CONVOLUTION OPERATION
$\left[V_{VI}(I)=\sum_{K=1}^{4}V_S(N_K\Delta t)\,h\left(\frac{t'-K\Delta t}{T}\right)\right]$
STORE $V_{IV}(I)$
SET $I=I+1$
IS $I<Bl_S$?
NO
EXIT
YES
READ NEXT t_R
TO FIG. 4a.
C

Fig. 4c.

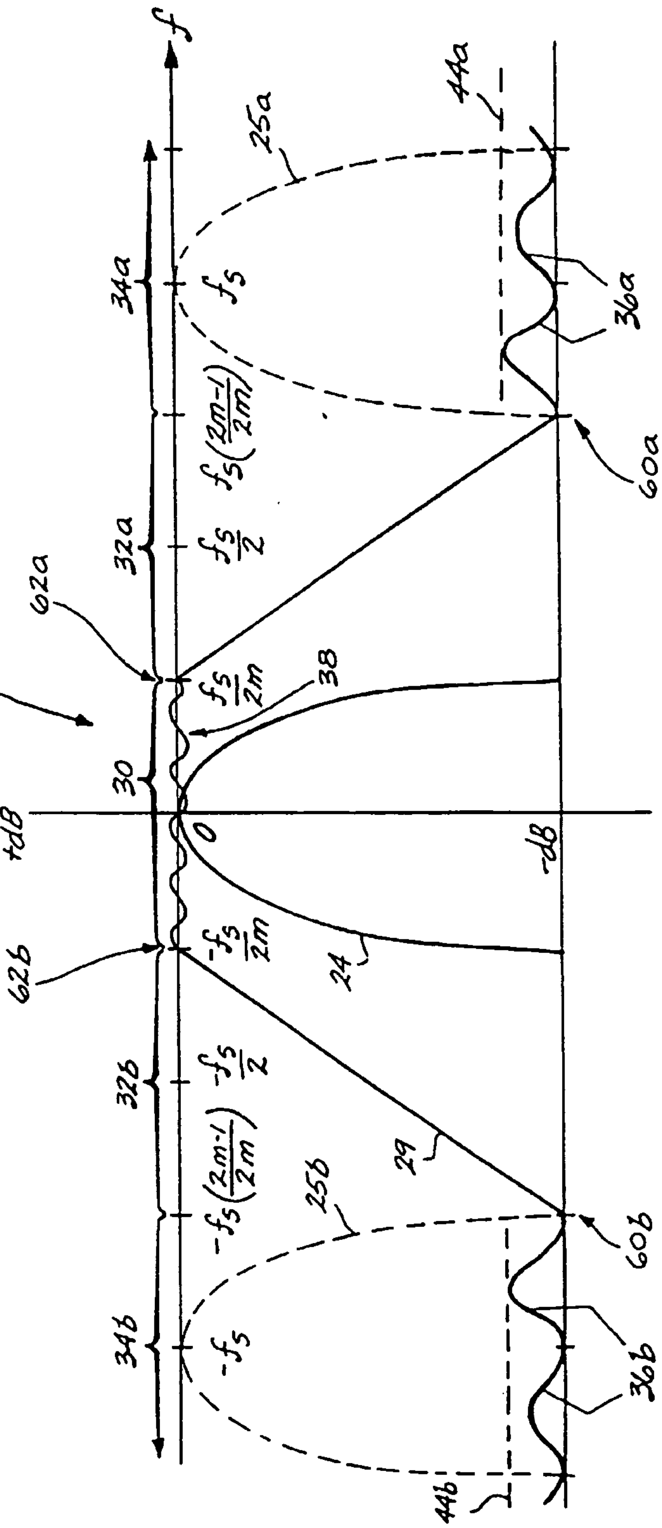
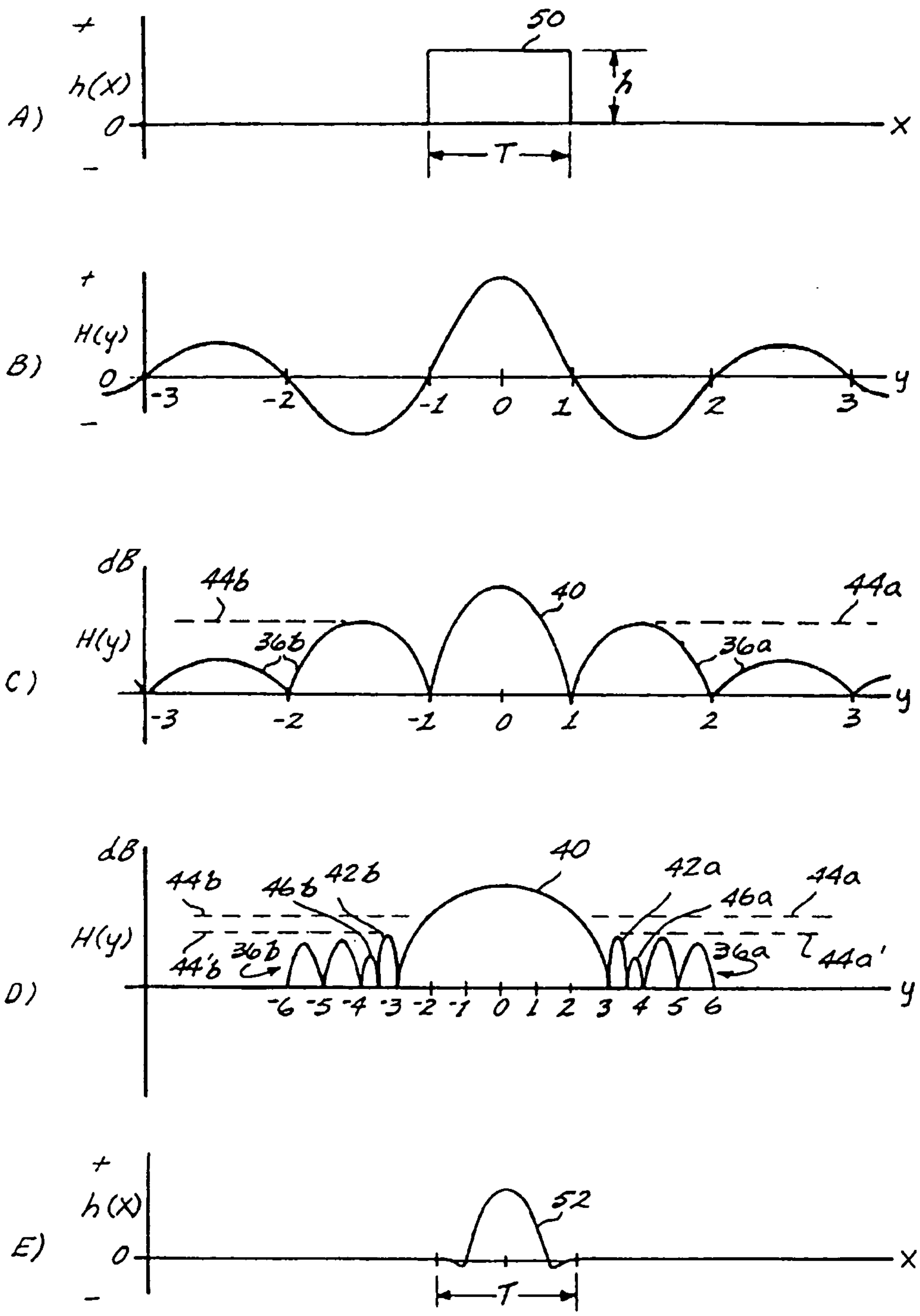

f
+dB
-dB
0
40
30
32a
32b
34a
34b
62a
62b
60a
60b
25a
25b
36a
36b
44a
44b
38
24
29
f_s
$-f_s$
$\frac{f_s}{2}$
$-\frac{f_s}{2}$
$\frac{f_s}{2m}$
$-\frac{f_s}{2m}$
$f_s\left(\frac{2m-1}{2m}\right)$
$-f_s\left(\frac{2m-1}{2m}\right)$

Fig. 5.



A)
+
h(X)
0
-
50
h
T
X
B)
+
H(y)
0
-
-3
-2
-1
0
1
2
3
y
C)
dB
H(y)
44b
36b
40
36a
44a
-3
-2
-1
0
1
2
3
y
D)
dB
H(y)
44b
46b
42b
40
42a
46a
44a
44'b
36b
36a
44a'
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
y
E)
+
h(X)
0
-
52
T
X

Fig. 6.

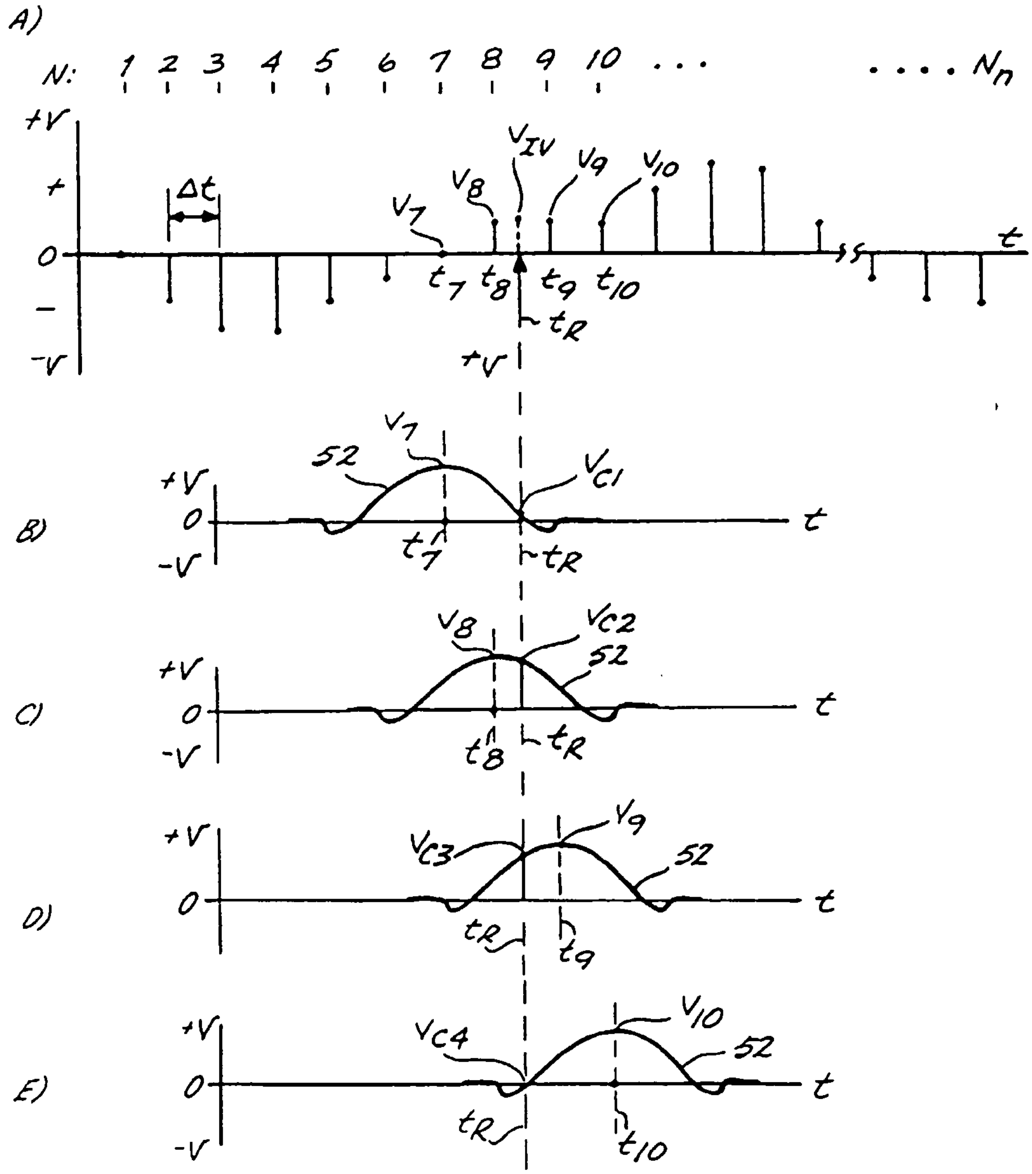

A)
N: 1 2 3 4 5 6 7 8 9 10 . . . . N_n
+V
+
0
-
-V
Δt
V_7
V_8
V_{IV}
V_9
V_{10}
t
t_7 t_8 t_9 t_{10}
t_R
+V
B)
52
V_7
V_{C1}
t_7
t_R
C)
V_8
V_{C2}
52
t_8
t_R
D)
V_{C3}
V_9
52
t_R
t_9
E)
V_{C4}
V_{10}
52
t_R
t_{10}

Fig. 7.